SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

Q.1 Let x_0 be the real number such that $e^{x_0} + x_0 = 0$. For a given real number α, define

$$g(x) = \frac{3[illegible] + 3[illegible] - [illegible] - [illegible]}{3([illegible] + 1)}$$

for all real numbers x.

Then which one of the following statements is TRUE?

(A)	For $\alpha = 2$, $\lim_{x \to x_0} \left\lvert \frac{g(x) + e^{x_0}}{x - x_0} \right\rvert = 0$
(B)	For $\alpha = 2$, $\lim_{x \to x_0} \left\lvert \frac{g(x) + e^{x_0}}{x - x_0} \right\rvert = 1$
(C)	For $\alpha = 3$, $\lim_{x \to x_0} \left\lvert \frac{g(x) + e^{x_0}}{x - x_0} \right\rvert = 0$
(D)	For $\alpha = 3$, $\lim_{x \to x_0} \left\lvert \frac{g(x) + e^{x_0}}{x - x_0} \right\rvert = \frac{2}{3}$

Q.2 Let $\mathbb{R}$ denote the set of all real numbers. Then the area of the region

$$\left\{(x, y) \in \mathbb{R} \times \mathbb{R} : x > 0, y > \frac{1}{x}, 5x - 4y - 1 > 0, 4x + 4y - 17 < 0\right\}$$

is

(A)	$\frac{17}{16} - \log_e 4$	(B)	$\frac{33}{8} - \log_e 4$
(C)	$\frac{57}{8} - \log_e 4$	(D)	$\frac{17}{2} - \log_e 4$

Q.3 The total number of real solutions of the equation

$$\theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right)$$

is

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\tan^{-1}x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, respectively.)

(A)	1	(B)	2	(C)	3	(D)	5

Q.4 Let S denote the locus of the point of intersection of the pair of lines

$$4x - 3y = 12\alpha\,,$$
$$4\alpha x + 3\alpha y = 12\,,$$

where α varies over the set of non-zero real numbers. Let T be the tangent to S passing through the points $(p, 0)$ and $(0, q)$, $q > 0$, and parallel to the line $4\,x - \frac{3}{\sqrt{2}}y = 0$.

Then the value of pq is

(A)	$-6\sqrt{2}$	(B)	$-3\sqrt{2}$	(C)	$-9\sqrt{2}$	(D)	$-12\sqrt{2}$

SECTION 2 (Maximum Marks: 16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme**:

  Full Marks : $+4$ **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : $+3$ If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : $+2$ If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : $+1$ If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get $+4$ marks;
  choosing ONLY (A) and (B) will get $+2$ marks;
  choosing ONLY (A) and (D) will get $+2$ marks;
  choosing ONLY (B) and (D) will get $+2$ marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get -2 marks.

Q.5 Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $P = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$. Let $Q = \begin{pmatrix} x & y \\ z & 4 \end{pmatrix}$ for some non-zero real numbers x, y, and z, for which there is a 2×2 matrix R with all entries being non-zero real numbers, such that $QR = RP$.

Then which of the following statements is (are) TRUE?

(A)	The determinant of $Q - 2I$ is zero
(B)	The determinant of $Q - 6I$ is 12
(C)	The determinant of $Q - 3I$ is 15
(D)	$yz = 2$

Q.6 Let S denote the locus of the mid-points of those chords of the parabola $y^2 = x$, such that the area of the region enclosed between the parabola and the chord is $\frac{4}{3}$. Let $\mathcal{R}$ denote the region lying in the first quadrant, enclosed by the parabola $y^2 = x$, the curve S, and the lines $x = 1$ and $x = 4$.

Then which of the following statements is (are) TRUE?

(A)	$(4, \sqrt{3}) \in S$
(B)	$(5, \sqrt{2}) \in S$
(C)	Area of $\mathcal{R}$ is $\frac{14}{3} - 2\sqrt{3}$
(D)	Area of $\mathcal{R}$ is $\frac{14}{3} - \sqrt{3}$

Q.7 Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two distinct points on the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

such that $y_1 > 0$, and $y_2 > 0$. Let C denote the circle $x^2 + y^2 = 9$, and M be the point $(3, 0)$.

Suppose the line $x = x_1$ intersects C at R, and the line $x = x_2$ intersects C at S, such that the y-coordinates of R and S are positive. Let $\angle ROM = \frac{\pi}{6}$ and $\angle SOM = \frac{\pi}{3}$, where O denotes the origin $(0, 0)$. Let $|XY|$ denote the length of the line segment XY.

Then which of the following statements is (are) TRUE?

(A)	The equation of the line joining P and Q is $2x + 3y = 3(1 + \sqrt{3})$
(B)	The equation of the line joining P and Q is $2x + y = 3(1 + \sqrt{3})$
(C)	If $N_2 = (x_2, 0)$, then $3\,|N_2Q| = 2\,|N_2S|$
(D)	If $N_1 = (x_1, 0)$, then $9\,|N_1P| = 4\,|N_1R|$

Q.8 Let $\mathbb{R}$ denote the set of all real numbers. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \frac{6x + \sin x}{2x + \sin x} & \text{if } x \neq 0, \\ \frac{7}{3} & \text{if } x = 0. \end{cases}$$

Then which of the following statements is (are) TRUE?

(A)	The point $x = 0$ is a point of local maxima of f
(B)	The point $x = 0$ is a point of local minima of f
(C)	Number of points of local maxima of f in the interval $[\pi, 6\pi]$ is 3
(D)	Number of points of local minima of f in the interval $[2\pi, 4\pi]$ is 1

SECTION 3 (Maximum Marks: 32)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:

Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;
Zero Marks : 0 In all other cases.

Q.9 Let $y(x)$ be the solution of the differential equation

$$[illegible]\frac{[illegible]}{dx} + [illegible] = [illegible] + [illegible], \quad x > \frac{1}{e},$$

satisfying $y(1) = 0$. Then the value of $2\frac{(y(e))^2}{y(e^2)}$ is **0.75** .

Q.10 Let $a_0, a_1, \dots, a_{23}$ be real numbers such that

$$\left(1 + \frac{2}{5}x\right)^{23} = \sum_{i=0}^{23} a_i x^i$$

for every real number x. Let a_r be the largest among the numbers a_j for $0 \le j \le 23$. Then the value of r is **6**

Q.11 A factory has a total of three manufacturing units, M_1, M_2, and M_3, which produce bulbs independent of each other. The units M_1, M_2, and M_3 produce bulbs in the proportions of $2:2:1$, respectively. It is known that 20% of the bulbs produced in the factory are defective. It is also known that, of all the bulbs produced by M_1, 15% are defective. Suppose that, if a randomly chosen bulb produced in the factory is found to be defective, the probability that it was produced by M_2 is $\frac{2}{5}$.

If a bulb is chosen randomly from the bulbs produced by M_3, then the probability that it is defective is is **0.3** .

Q.12 Consider the vectors

$$\vec{x} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \quad \vec{y} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}, \quad \text{and} \quad \vec{z} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}.$$

For two distinct positive real numbers α and β, define

$$\vec{X} = \alpha\vec{x} + \beta\vec{y} - \vec{z}, \quad \vec{Y} = \alpha\vec{y} + \beta\vec{z} - \vec{x} \quad \text{and} \quad \vec{Z} = \alpha\vec{z} + \beta\vec{x} - \vec{y}$$

If the vectors $\vec{X}, \vec{Y}$, and $\vec{Z}$ lie in a plane, then the value of $\alpha + \beta - 3$ is is **–2**

Q.13 For a non-zero complex number z, let $\arg(z)$ denote the principal argument of z, with $-\pi < \arg(z) \leq \pi$. Let ω be the cube root of unity for which $0 < \arg(\omega) < \pi$. Let

$$z = \arg\left(\sum_{k=1}^{2025} (-\omega)^k\right).$$

Then the value of $-\frac{3z}{\pi}$ is **–2** .

Q.14 Let $\mathbb{R}$ denote the set of all real numbers. Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to (0, 4)$ be functions defined by

$$f(x) = \log_e(x^2 + 2x + 4), \quad \text{and} \quad g(x) = \frac{4}{1 + e^{-2x}}.$$

Define the composite function $f \circ g^{-1}$ by $(f \circ g^{-1})(x) = f(g^{-1}(x))$, where g^{-1} is the inverse of the function g.

Then the value of the derivative of the composite function $f \circ g^{-1}$ at $x = 2$ is **0.25**

Q.15 Let

$$\alpha = \frac{1}{\sin 60^\circ \sin 61^\circ} + \frac{1}{\sin 62^\circ \sin 63^\circ} + \cdots + \frac{1}{\sin 118^\circ \sin 119^\circ}.$$

Then the value of

$$\left(\frac{\operatorname{cosec} 1^\circ}{\alpha}\right)^2$$

is **3**

Q.16 If

$$I = \int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{2x^2 - 3x + 2}\, dx,$$

then the value of $\sqrt{7} \tan\left(\frac{2I\sqrt{7}}{\pi}\right)$ is **21** .

(Here, the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.)

END OF THE QUESTION PAPER